Post-critical $SsPmp$ and its applications to Virtual Deep Seismic Sounding (VDSS) – 2: 1-D imaging of the crust/mantle and joint constraints with receiver functions

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1 INTRODUCTION

In recent years, Virtual Deep Seismic Sounding (VDSS) has been used to measure Moho depth in different areas (e.g. Tseng et al. 2009; Yu et al. 2012, 2016; Tian et al. 2015; Kang et al. 2016; Parker et al. 2016; Thompson et al. 2019). $SsPmp$, the seismic phase used in VDSS, originates when upcoming teleseismic $S$ waves convert to downgoing $P$ waves at the free surface (the virtual source), which then undergo post-critical reflection at or within the crust–mantle boundary (CMB). Here, we synthesize our methodology of deriving Moho depth, average crustal $V_p$ and uppermost-mantle $V_p$ from single-station observations of post-critical $SsPmp$ under a 1-D assumption. We first verify our method with synthetics and then substantiate it with a case study using the Yellowknife and POLARIS arrays in the Slave Craton, Canada. We show good agreement of crustal and upper-mantle properties derived with VDSS with those given by previous active-source experiments and our own $P$ receiver functions (PRF) in our study area. Finally, we propose a PRF-VDSS joint analysis method to constrain average crustal $V_p/V_s$ ratio and composition. Our PRF-VDSS joint analysis shows that the southwest Slave Craton has an intermediate crustal composition, most consistent with a Mesoarchean age.

Key words: Body waves; Composition and structure of the continental crust; Cratons; Crustal imaging; North America; Wave scattering and diffraction.
by CMB depth and $V_{p}^{\mu}$, $\Phi_{\text{VDSS}}$ is sensitive to lower-crustal and uppermost-mantle $V_{p}$ ($V_{p}^{lc}$ and $V_{p}^{um}$), which implies the possibility of constraining both $V_{p}^{lc}$ and $V_{p}^{um}$ with observations of $\Phi_{\text{VDSS}}$.

Here, we first show that we can estimate $T_{\text{VDSS}}$ by picking peaks on $SsPmp$ envelope functions and use it to derive Moho depth (Parker et al. 2016). We then present synthetic tests to demonstrate that $V_{p}^{um}$ can be constrained without a priori knowledge of $V_{p}^{lc}$ using post-critical $SsPmp$ with turning velocity $1/p$ close to $V_{p}^{um}$. In the presence of noise, we show that the estimation of $V_{p}^{um}$ can be significantly improved by incorporating $\Phi_{\text{VDSS}}$ measurements from multiple events. In addition, we propose a joint PRF-VDSS analysis scheme that simultaneously constrains $V_{p}$ and average crustal $V_{p}/V_{s}$ ratio, thus crustal average composition ($V_{p}/V_{s}$ ratio is also denoted $\kappa$, particularly when used in context of the $H-\kappa$ analysis of receiver functions: Zhu & Kanamori 2000). Finally, we validate our proposed methods using the Yellowknife and POLARIS arrays in the SW Slave Craton, finding good agreement between the crustal and upper-mantle properties we derive with VDSS and those from previous studies using active-source experiments and our own $P$ receiver functions (PRF). We demonstrate with our PRF-VDSS joint analysis that the SW Slave Craton has an intermediate crustal composition.

### 2 Synthetic Examples

#### 2.1 Measuring $T_{\text{VDSS}}$ from envelope functions

When a signal undergoes a phase shift, its envelope function stays constant despite the changing waveform, (Aki & Richards 2002a). Therefore, a simple way to account for the phase shift of $SsPmp$
relative to $S_s$ while measuring $T_{\text{VDSS}}$ to calculate the time between the peaks on $S_s$ and $SsPmp$ envelope functions (Parker et al. 2016). Fig. 2(a) shows synthetic waveforms calculated using the reflectivity algorithm (Randall 1989) for the 1-D models in Fig. 1 (hereafter ‘Model #1’) and $0.124 \leq \rho \leq 0.140$ km s$^{-1}$, after separation into pseudo-$P$ (motion associated with incoming $P$ waves) and pseudo-$S$ (motion associated with incoming $S$ waves) components (hereafter referred to as $P$ and $S$ for simplicity) with a particle-motion analysis algorithm (Yu et al. 2013) and converting to envelope functions. The shapes of $S_s$ ($S$ component) and $SsPmp$ ($P$ component) envelope functions are essentially identical, as the arrivals are only distinguished by a phase shift ($\Phi_1$) and/or envelope functions. The shapes of $S_s$ and $SsPmp$ envelope functions and convert them to Moho depths using eq. (1) and the true $V_p^\text{av}$. The estimated Moho depths agree (within 0.1 km) with the true CMB depth for all $p$ (Fig. 2b), indicating the robustness of this method. Given multiple $SsPmp$ observations with a wide range of ray parameter $p$, one can simultaneously determine Moho depth and $V_p^\text{av}$ (Kang et al. 2016), as shown below. Although $T_{\text{VDSS}}$ can be robustly estimated from $SsPmp$ envelope functions, when calculating these envelope functions we effectively eliminate the information contained in $\Phi_{\text{VDSS}}$ (Fig. 2a), which is sensitive to the structure of the CMB (Part 1). Therefore, a more desirable way to analyse $SsPmp$ is to model $T_{\text{VDSS}}$ and $\Phi_{\text{VDSS}}$ simultaneously.

2.2 Modeling $T_{\text{VDSS}}$ and $\Phi_{\text{VDSS}}$ simultaneously for a single event

In order to model $T_{\text{VDSS}}$ and $\Phi_{\text{VDSS}}$ simultaneously, we first generate synthetic $SsPmp$ waveforms for all combinations of $V_p$ in the lower crust ($V_p^\text{lc}$) and upper mantle ($V_p^\text{um}$) using the reflection coefficient at the Moho (Aki & Richards 2002b). We then use cross-correlation to align the synthetic $SsPmp$ waveforms with the ‘observed’ data (e.g. the waveforms in Fig. 1b) and compute the normalized misfit (hereafter referred to as misfit for simplicity) between the synthetic and ‘observed’ $SsPmp$. Before computing the misfit, we normalize the ‘observed’ and synthetic $SsPmp$ by the peak values of their envelope functions so that the effects of amplitude difference are eliminated. The synthetic $SsPmp$ waveform with the minimum misfit determines the best-fitting $V_p^\text{lc}$ and $V_p^\text{um}$, and the best-fitting $T_{\text{VDSS}}$ is found by picking the peak cross-correlation value between the best-fitting synthetic $SsPmp$ and the ‘observed’ one. We note that previous 1-D modelling schemes have held $V_p^\text{lc}$ and/or $V_p^\text{um}$ fixed (Tseng et al. 2009; Tian et al. 2015; Yu et al. 2016). To test our approach, we use the reflectivity method (Randall 1989) to compute ‘observed’ waveforms and try to recover $T_{\text{VDSS}}, V_p^\text{lc}$ and $V_p^\text{um}$ from them using the method described above.

We first attempt to recover both $V_p^\text{lc}$ and $V_p^\text{um}$ from the observed $SsPmp$ waveforms. Our ‘observed’ waveforms are computed with Model #1, a single layer crust with $V_p = 6.5$ km s$^{-1}$ overlaying a half space with $V_p = 8.1$ km s$^{-1}$ that represents the upper mantle (Fig. 1a). We use two different ray parameters $p = 0.128$ and 0.136 km s$^{-1}$ to study the potential effects of ray parameter. After applying our proposed method, the resulting misfits are plotted as functions of $V_p^\text{lc}$ and $V_p^\text{um}$ (Fig. 3). In the case with $p = 0.136$ km s$^{-1}$ ($1/p = 7.35$ km s$^{-1}$), we observe that the misfit depends strongly on $V_p^\text{um}$ and weakly on $V_p^\text{lc}$, with the misfit contours having small negative slopes (negative trade-off between $V_p^\text{lc}$ and $V_p^\text{um}$; Fig. 3a). In contrast, in the case with $p = 0.136$ km s$^{-1}$ ($1/p = 7.35$ km s$^{-1}$), although the trade-off between $V_p^\text{lc}$ and $V_p^\text{um}$ is still negative, the misfit depends weakly on both $V_p^\text{lc}$ and $V_p^\text{um}$ (Fig. 3b). [Note that because the observed and synthetic waveforms are not computed in the same way, minor differences exist between them that makes the minimum misfits in both cases non-zero (Fig. 3).] Because fitting $SsPmp$ waveforms is equivalent to fitting $\Phi_{\text{VDSS}}$, this behaviour of misfit can be understood by considering the dependence of $\Phi_{\text{VDSS}}$ on $V_p^\text{lc}$ and $V_p^\text{um}$. When $1/p$ is close to $V_p^\text{lc}$, for example the case with $p = 0.128$ km s$^{-1}$ ($1/p = 7.81$ km s$^{-1}$; Fig. 3a), $\Phi_{\text{VDSS}}$ is primarily controlled by $V_p^\text{um}$ with only weak dependence on $V_p^\text{lc}$ (Fig. 7a in Part 1). When $1/p$ is close to neither $V_p^\text{lc}$ nor $V_p^\text{um}$, for example the case with $p = 0.136$ km s$^{-1}$ ($1/p = 7.35$ km s$^{-1}$; Fig. 3b), $\Phi_{\text{VDSS}}$ depends weakly on both $V_p^\text{lc}$ and $V_p^\text{um}$ (Part 1). Ideally, when $1/p$ is close to $V_p^\text{lc}$, misfit will be primarily controlled by $V_p^\text{lc}$ due to the strong dependence of $\Phi_{\text{VDSS}}$ on $V_p^\text{lc}$ (Fig. 7b in Part 1). However, in this case the large $p$ may cause strong pre-critical reflections from intracrustal interfaces that may interfere with $SsPmp$ and distort its waveform (e.g. Fig. 5 in Part 1). Therefore, a more practical way to utilize $\Phi_{\text{VDSS}}$ observations is to infer $V_p^\text{um}$ from $\Phi_{\text{VDSS}}$ while measuring $V_p^\text{lc}$ in cases with $1/p$ close to $V_p^\text{um}$, because in such cases an incorrect $V_p^\text{lc}$ would only have limited effect on estimated $V_p^\text{um}$ (Fig. 3a).

Fig. 4 shows examples of this strategy of estimating Moho depth and $V_p^\text{um}$ from $SsPmp$ waveforms while fixing $V_p^\text{lc}$. We use the correct $V_p^\text{um} = 6.5$ km s$^{-1}$ in eq. (1) to convert the measured $T_{\text{VDSS}}$ to Moho depth. For $p = 0.128$ km s$^{-1}$, when the true $V_p^\text{lc} = 6.5$ km s$^{-1}$ ($V_p^\text{lc}$ and $V_p^\text{av}$ are the same here because the crust is homogeneous) is assumed, we are able to recover both the CMB depth and $V_p^\text{um}$ precisely (Fig. 4a). Because we precisely recovered $V_p^\text{um}$ and $V_p^\text{lc}$, the synthetic $SsPmp$ matches the observation almost perfectly (Fig. 4b). For the same ‘observed’ data ($p = 0.128$ km s$^{-1}$), when we vary $V_p^\text{lc}$ from the true value to 6.2 and 6.8 km s$^{-1}$ (±5 per cent perturbation), only ±0.6 per cent error is introduced in estimated $V_p^\text{um}$ ($=0.05$ km s$^{-1}$), and its effect on estimated Moho depth is negligible (Fig. 4c). The misfit curves for $V_p^\text{lc} = 6.2, 6.5$ and 6.8 km s$^{-1}$ have very similar minima, indicating that the synthetics fit the ‘observed’ data equally well for all three cases (Fig. 4d). Because $\Phi_{\text{VDSS}}$ is matched precisely in all three cases, the correct $T_{\text{VDSS}}$ is found in each case and we are able to give the correct CMB depth despite errors in assumed $V_p^\text{lc}$ (Fig. 4c). However, when $p = 0.136$ km s$^{-1}$, again varying $V_p^\text{lc}$ from 6.2 to 6.8 km s$^{-1}$ causes $\sim$1–2 per cent error in the estimated $V_p^\text{um}$, significantly larger than for $p = 0.128$ km s$^{-1}$, though the effect on estimated Moho depth remains negligible (Fig. 4e) due to precise matching of the $SsPmp$ waveforms. We also observe that the troughs of the misfit curves are significantly wider for $p = 0.136$ km s$^{-1}$ than for $p = 0.128$ km s$^{-1}$, indicating poorer constraints on $V_p^\text{um}$ (Fig. 4f).

These results show that, in a 1-D earth, matching $\Phi_{\text{VDSS}}$ alone is sufficient to estimate $T_{\text{VDSS}}$ and Moho depth precisely, whereas to infer $V_p^\text{um}$ from $\Phi_{\text{VDSS}}$ requires a priori knowledge of $V_p^\text{lc}$. If $V_p^\text{lc}$ is poorly constrained in the study region, it is better to use $SsPmp$ waveforms with $1/p$ close to $V_p^\text{um}$, so that an incorrectly assumed $V_p^\text{lc}$ causes less error in the estimated $V_p^\text{um}$. Even in cases with well-constrained $V_p^\text{lc}$, using $SsPmp$ waveforms with $1/p$ close to $V_p^\text{um}$ is still preferred because the narrower troughs of the misfit curves in such cases would result in better constrained $V_p^\text{um}$ (smaller uncertainty; Supplementary Text 1).

Because in field data the $S$ waves travel are usually contaminated by $P$-wave coda, it is important to test our methods for cases with reasonable noise level. To simulate realistic noise, we generate Gaussian white noise, bandpass filter it between 0.05 and 0.5 Hz (a range typically used for filtering field data) and scale the noise
To demonstrate this, we generate 100 random simulations of noisy $S_{sPmp}$ waveforms used in the noise-free case (Fig. 4) and again attempt to retrieve $V_{p}^{um}$ and Moho depth (Fig. S1). For $p = 0.128$ s km$^{-1}$, the derived $V_{p}^{um}$ and Moho depth are significantly biased due to the interference of noise with the $S_{sPmp}$ waveform (Figs S1a–d), whereas the two parameters are reasonably well-recovered for the case with $p = 0.136$ s km$^{-1}$ (Figs S1e and f). Since the added noise is random, the higher bias for the $p = 0.128$ s km$^{-1}$ case is likely a coincidence (noise level happens to be higher in the time window of $S_{sPmp}$). To demonstrate this, we generate 100 random simulations of noisy $S_{sPmp}$ both for $p = 0.128$ s km$^{-1}$ and for $p = 0.136$ s km$^{-1}$ and derive $V_{p}^{um}$ from each simulation (Fig. S2). The mean estimated $V_{p}^{um}$ from 100 simulations (equivalent to 100 earthquakes with the same $p$ recorded at a single station) are consistent with the true value both for $p = 0.128$ s km$^{-1}$ and for $p = 0.136$ s km$^{-1}$, even though individual simulations (single earthquakes) can produce significant errors (Fig. S2). Therefore, the larger error for $p = 0.128$ s km$^{-1}$ in Fig. S1 is indeed coincidental. Estimated $V_{p}^{um}$ has a larger standard deviation for $p = 0.136$ s km$^{-1}$ than for $p = 0.128$ s km$^{-1}$ (Fig. S2) due to higher sensitivity of $\Phi_{VDSS}$ to $V_{p}^{um}$ when $p = 0.128$ s km$^{-1}$ (Figs 4 and S1). In order to avoid these errors due to random noise, an obvious strategy is to combine observations of post-critical $S_{sPmp}$ from multiple events in our estimation of Moho depth and $V_{p}^{um}$.

2.3 Crustal and mantle properties from $T_{VDSS}$ and $\Phi_{VDSS}$ of multiple events

To incorporate observations of multiple events, we first measure $T_{VDSS}$ and $\Phi_{VDSS}$ from individual events and then estimate crustal and mantle properties by modeling the observed $T_{VDSS}$ and $\Phi_{VDSS}$. In order to measure $T_{VDSS}$ and $\Phi_{VDSS}$ simultaneously, for each event we use a cosine-tapered 15-s window around the $S_{s}$ waveform on the $S$ component as the source wavelet of that event. We then apply phase shifts from $0^\circ$–$360^\circ$ to the source wavelet to create synthetic $S_{sPmp}$ waveforms with all possible phase shifts. We use cross-correlation to find the best alignment between each synthetic $S_{sPmp}$ wavelet and the observed $S_{sPmp}$. We then normalize observed and synthetic $S_{sPmp}$ and compute the misfit between them. $\Phi_{VDSS}$ is determined as the phase shift that minimizes the misfit between the synthetic and modeled $S_{sPmp}$. We estimate the uncertainty of $T_{VDSS}$ and $\Phi_{VDSS}$ from the curvature of the misfit-phase-shift function around $\Phi_{VDSS}$ (Text S1 and Fig. S3). We then find $T_{VDSS}$ of each event by cross-correlation between the best-fit synthetic $S_{sPmp}$ and the observed $S_{sPmp}$. After measuring $T_{VDSS}$ and $\Phi_{VDSS}$ for all events, we use the two parameters as functions of ray parameters to constrain crustal and mantle properties. Following Kang et al. (2016), we use $T_{VDSS}$ as a function of $p$ (the moveout of post-critical $S_{sPmp}$) to constrain $V_{p}^{lc}$ and Moho depth $H$ with linear regression, which also gives uncertainties of estimated $V_{p}^{um}$ and $H$ (Fig. 5c). In our linear regression procedure, we scale each data point with its uncertainty, so that data points with high uncertainty have smaller weight on the results. To constrain $V_{p}^{um}$, we compare observed $\Phi_{VDSS}$ as a function of $p$ with theoretical $\Phi_{VDSS}$ relations computed with a fixed $V_{p}^{lc}$ and a range of $V_{p}^{um}$ (Fig. 5d).

To test our method, we generate synthetic $S_{sPmp}$ waveforms using Model #1 and $p = 0.124–0.134$ s km$^{-1}$, on which we can clearly observe a decrease in $T_{VDSS}$ and $\Phi_{VDSS}$ with increasing $p$ (Fig. 5a). We then measure $T_{VDSS}$ and $\Phi_{VDSS}$ from the observed $S_{sPmp}$ of each event, an example of which is shown in Fig. 5(b). The observed $T_{VDSS}$-$p$ relation gives a best-fitting $V_{p}^{av} = 6.5 \pm 0.1$ km s$^{-1}$ and $H = 40 \pm 1$ km, in good agreement with the input model (Fig. 5c). To find the best-fitting $V_{p}^{um}$, we plot theoretical $\Phi_{VDSS}$-$p$ relations with different $V_{p}^{um}$ values while assuming $V_{p}^{lc} = 6.5$ km s$^{-1}$, the true $V_{p}^{lc}$, and compare them with the observed values. The comparison shows that the observations favour $V_{p}^{um} = 8.1$ km s$^{-1}$, consistent with the input model (Fig. 5d). To test the effects of assumed $V_{p}^{lc}$ on the estimated $V_{p}^{um}$, we compare theoretical curves computed assuming $V_{p}^{lc} = 6.2$, $6.5$, $6.8$ km s$^{-1}$ ($\sim$5 per cent perturbation; Fig. 6). For $V_{p}^{lc} = 6.2$ and $6.8$ km s$^{-1}$, the theoretical $\Phi_{VDSS}$-$p$ curves overlap when $p$ is small and gradually diverge as $p$ increases (Fig. 6), but never to the extent that would significantly affect the estimated $V_{p}^{um}$ (Fig. 6). The theoretical $\Phi_{VDSS}$-$p$ relations are insensitive to assumed $V_{p}^{lc}$ because the reciprocal of ray parameter ($1/p$) considered here ($7.5–8.1$ km s$^{-1}$) is significantly higher than the assumed $V_{p}^{lc}$, which makes $\Phi_{VDSS}$ insensitive to $V_{p}^{lc}$ (Fig. 7a in Part I). In the case of $V_{p}^{lc} = 6.2$ km s$^{-1}$ ($6.8$ km s$^{-1}$), the theoretical $\Phi_{VDSS}$-$p$ curves are shifted slightly upward (downward) compared with $V_{p}^{lc} = 6.5$ km s$^{-1}$, because for a fixed $p$, decreasing (increasing) $V_{p}^{lc}$ while fixing $\Phi_{VDSS}$ requires an increase (decrease) in $V_{p}^{um}$ (negative trade-off between $V_{p}^{lc}$ and $V_{p}^{um}$; Fig. 7 in Part I).
Figure 4. Estimating Moho depth and $V_p^{um}$ from waveforms computed for Model #1 with an assumed $V_p^{lc}$. (a) and (b) Moho depth and $V_p^{um}$ derived using the correct $V_p^{lc} = 6.5$ km s$^{-1}$. (a) Gray dashed lines: Moho depth and $V_p^{um}$ that best fit $T_{VDDS}$ and $\Phi_1$ very closely matching Model #1 (black line). $V_{av} = 6.5$ km s$^{-1}$ is used to convert $T_{VDDS}$ to Moho depth. (b) Synthetic $P$ and $S$ waveforms computed for Model #1 and $p = 0.128$ s km$^{-1}$ ('observations'). Grey dashed curve: $P$-component $StPmp$ waveform that best fits $T_{VDDS}$ and $\Phi_1$. (c–f) A comparison of results derived with different assumed $V_p^{lc}$. (c) and (e) Model #1 as in (a) with Moho depths and $V_p^{um}$ estimated for assumed $V_p^{lc} = 6.2$ (red), 6.5 (grey) and 6.8 (blue) km s$^{-1}$, for $p = 0.128$ and 0.136 s km$^{-1}$ respectively. Black dashed lines mark the critical velocities ($1/p$) in the two cases. (d) and (f) Normalized misfits as functions of $V_p^{um}$ for each case. Note the choice of $V_p^{lc}$ has little effect on estimated $V_p^{um}$ when $p$ is small (1/p is large and close to $V_p^{um}$), but has a significant effect when $p$ is large (1/p is small and far from $V_p^{um}$).
We also test our multi-event method using noisy synthetic data. We contaminate our synthetic data with the same 10 per cent band-limited white noise as described before (Fig. S4a). Whereas SsPmp is clearly observed despite the addition of noise, Sp, the phase used in S receiver functions, is barely observable (Fig. S4a). We then process the noisy synthetic waveforms as described above. The measured $T_{VDSS}$ and $\Phi_{VDSS}$ have significantly higher uncertainties compared to the noise-free case (0.2 s versus 0.1 s and 14° versus 8° for the event with $p = 0.126$ s·km⁻¹; Fig. S4b). Nonetheless, our method still yields $V_p^{av}$ and $H$ that agree reasonably well with the input model ($6.4 \pm 0.2$ km·s⁻¹ versus 6.5 km·s⁻¹ and 38 ± 3 km versus 40 km; Fig. S4c). The observed $\Phi_{VDSS}$-$p$ relation also favours $V_p^{um} = 8.1 \pm 0.1$ km·s⁻¹ despite scattering of the data points (Fig. S4d). In short, we show that our proposed multi-event analysis can robustly retrieve $V_p^{av}$, $H$ and $V_p^{um}$ from data with moderate noise levels, making it suitable for application to field data.

2.4 Average crustal $V_p/V_s$ ratio ($\kappa$) and composition from joint PRF-VDSS analysis

Because rock $V_p/V_s$ ratios are sensitive to rock composition (Christensen 1996), estimation of average crustal $V_p/V_s$ ratio (also denoted $\kappa$) can offer insights into bulk composition of the crust, thus tectonic evolution of the area. Conventionally, $\kappa$ has been estimated using the $H$-$\kappa$ method, which searches for the model that maximizes the stacked energy of Moho $Ps$ and multiples on PRFs ($PP_{Pms}$, $PP_{Sms}$, etc.; Zhu & Kanamori 2000). However, the conventional $H$-$\kappa$ method has two major disadvantages. First, it uses the Moho multiples on PRF, which are not always robustly observed. Secondly,
it requires \( V_p^{av} \) as an input parameter, which needs to be estimated independently. A recent proposal to complement conventional \( H - \kappa \) method with \( SsPmp \) observations in order to estimate \( V_p^{av} \) still requires robust observation of at least one Moho multiples on PRFs (Luo et al. 2018). Here, we present a joint PRF-VDSS analysis method to derive \( \kappa \) without this limitation by using the moveout of \( SsPmp \) to constrain \( V_p^{av} \) and \( H \).

With a 1-D assumption, the arrival time of the Moho \( Ps \) in PRF can be expressed as:

\[
T_{Ps} = H \left( \sqrt{\frac{k}{V_p^{av}}} - p^2 - \sqrt{\frac{1}{V_p^{av}} - p^2} \right)
\]

Using \( H \) (hereafter \( H_{VDSS} \)) and \( V_p^{av} \) given by \( SsPmp \) observations, we can derive \( \kappa \) (hereafter \( \kappa_{joint} \)) from \( T_{Ps} \) (Fig. 7). To estimate the uncertainty of our \( \kappa_{joint} \), we draw 5000 random samples of \( V_p^{av} \) and \( H_{VDSS} \) from their joint distribution (Kang et al. 2016) and compute \( \kappa_{joint} \) for each pair. We then estimate uncertainty of our \( \kappa_{joint} \) from the variance of the 5000 randomly simulated \( \kappa_{joint} \) (Fig. 7). Since \( V_p^{av} \) and \( \kappa_{joint} \) together place key constraints on average crustal composition (Christensen & Mooney 1995; Christensen 1996), we also compute the joint distribution of \( V_p^{av} \) and \( \kappa_{joint} \) and compare them with laboratory measurements (Fig. 8).

We again use synthetic data to test our method. We generate synthetic PRFs using Model #1, apply normal moveout, and stack them. From the stacked PRF, the Moho \( T_{Ps} \) is picked at 4.50 s (Fig. 7a). Using our \( T_{Ps} \) and the previously estimated \( V_p^{av} = 6.5 \pm 0.1 \text{ km s}^{-1} \) and \( H_{VDSS} = 40 \pm 1 \text{ km} \), we find \( \kappa_{joint} = 1.73 \pm 0.02 \) (Fig. 7b), consistent with the input model. We also plot the joint distribution of \( V_p^{av} \) and \( \kappa_{joint} \), which shows a clear negative correlation between the two parameters (Fig. 8). To compare our seismic ‘observations’ with laboratory measurements, we plot \( V_p \) and \( \kappa \) of major crustal rock types measured at 600 MPa (corresponds to mid-crustal depth) and room temperature from Christensen (1996, Fig. 8). We also plot \( V_p \), \( \kappa \) and SiO\(_2\) content of individual samples (excluding eclogites and ultramafic rocks) measured at 600 MPa and room temperature (compilation of Hacker et al. 2015). For typical crustal rocks, \( \frac{dV_p}{dT} \approx -0.4 \text{ m s}^{-1} \text{ K}^{-1} \), and \( \frac{dV}{dT} \approx -0.2 \text{ m s}^{-1} \text{ K}^{-1} \), so typical \( \frac{d\kappa}{dT} \approx -0.0015/100 \text{ K} \) (Christensen & Mooney 1995; Barruol & Kern 1996; Christensen 1996). With these relations, we apply temperature corrections to the measurements from Christensen (1996) and Hacker et al. (2015) assuming a mid-crustal temperature of \( \sim 250^\circ \text{C} \), suitable for cratonic crust. We note that the temperature correction (\( -0.1 \text{ km s}^{-1} \)) is moderate for \( V_p \), but negligible (\( -0.004 \)) for \( \kappa \), indicating that \( \kappa \) is primarily controlled by rock composition (Christensen 1996). The laboratory measurements show a clear correlation of increasing \( V_p \) and \( \kappa \) with decreasing SiO\(_2\) content (Fig. 8). Because our \( V_p - \kappa \) correlation from VDSS-PRF analysis is orthogonal to the \( V_p - \kappa \) correlation of laboratory measurements, our method constrains the average crustal composition in this synthetic example as intermediate (equivalent to diorite or felsic granulite, Fig. 8). We recognize the large scatter of individual laboratory measurements of similar composition (coloured circles; Hacker et al. 2015) and the large variation between aggregated samples that share the same rock name (diamonds; Christensen 1996) around the positive \( V_p - \kappa \) trend, but note that these compilations represent samples collected in highly diverse geological settings. When applying our method to a specific region, a more desirable approach would be to build a rock physics model appropriate for that region.

### 3 OBSERVATIONS AND INTERPRETATIONS OF THE SOUTHWEST SLAVE CRATON

The Canadian Shield has a longevity and stability suggestive of a lithospheric structure closer to 1-D than tectonically active areas. The Yellowknife Array was deployed in the Slave Craton of the Canadian Shield (Figs 9a and b) in 1962 and was upgraded to digital recording in 1989 (Bostock 1998). The long deployment time and its location at the core of the Canadian Shield makes it an ideal place to test our proposed method for retrieving and analysing both \( T_{VDSS} \) and \( \Phi_{VDSS} \) under a 1-D assumption. In addition, the LITHOPROBE project acquired wide-angle refraction and near-vertical reflection profiles through Yellowknife (Fernández-Viejo & Clowes 2003; Fernández-Viejo et al. 2005; Hammer et al. 2010), allowing direct comparison between VDSS and active-source results.

#### 3.1 VDSS and PRF data analysis

Among stations of the Yellowknife Array, YKW3 has the longest archived broadband recording (1994–2014). In addition, Station EDZN of the POLARIS array (Snyder & Brunet 2007) was located \( \sim 80 \text{ km NW} \) of YKW3, within \( \sim 25 \text{ km of } SsPmp \) Moho reflection points for Kamchatka-Kurile earthquakes recorded at YKW3 (Fig. 9b), allowing us to compare PRF results at EDZN with VDSS results at YKW3, as conversion points of PRFs at Moho depth are typically \( < 20 \text{ km away from the station} \). For VDSS analysis at YKW3, we examined 56 teleseismic events in the epicentral range 30–60° with the back-azimuth range of 290–310°. We choose this narrow backazimuth range to avoid possible complications from lateral variation in lithospheric structure [the Moho reflection points of our selected events are all within 25 km of each other (Fig. 9b)], while still including plenty of events from the Kamchatka subduction zone with a wide range of ray parameters (Figs 9a and c). We removed instrumental response and applied a bandpass Butterworth filter between 0.05 and 0.5 Hz. After separating radial and vertical-component data into \( P \) and \( S \) components, we inspected the traces and their particle-motion diagrams, selecting the 19 events with simple \( Ss \) waveforms and significant \( P \) energy following the \( S \) arrival. We next computed the envelope functions of the \( P \) and \( S \) components of the 19 events and rejected the nine ‘Grade C’ events for which the ratio of the maxima of the \( P \) and \( S \) envelope functions \( < 0.6 \). The remaining 10 events all have simple \( Ss \) waveforms and...
Figure 7. Synthetic example of constraining average crustal $V_p/V_s$ ratio ($\kappa$) via PRF-VDSS joint analysis. (a) Synthetic PRF computed with Model #1, after moveout-correction to normal incidence. (b) Determination of crustal average $V_p/V_s$ ratio ($\kappa_{\text{joint}}$) from PRF $T_{Ps}$, VDSS Moho depth ($H_{\text{VDSS}}$), and VDSS average crustal $V_p(V_p^{av})$. Dashed black lines mark the estimated uncertainty of $\kappa_{\text{joint}}$, which is computed by taking the standard deviation of 5000 random simulations of $\kappa_{\text{joint}}$ (cyan histogram). Note $\kappa$ is well recovered.

Figure 8. Synthetic example of constraining average crustal composition with PRF-VDSS joint analysis. The probability density function is derived from 5000 random simulations of $V_p^{av}$ and $\kappa$. Circles coloured by per cent SiO$_2$ are measurements for crustal rock samples (felsic to mafic) at 600 MPa and room temperature (compilation of Hacker et al. 2015), which are then corrected to 250 °C (see Section 2.4). Gray diamonds and data ranges are measurements at 600 MPa and room temperature from Christensen (1996), which are also corrected to 250 °C. GAB: Gabbro-norite-troctolite; MGR: Mafic granulite; DIO: Diorite; FGR: Felsic granulite; GGN: Granite gneiss; GRA: Granite-granodiorite; AMP: Amphibolite. Note the orthogonal trends of the probability density function and per cent SiO$_2$, which constrains average crustal composition.

strong $SsPmp$ following the $Ss$ arrival (Fig. 9c). When sorted by their ray parameter, the 10 traces show a clear decrease of $T_{\text{VDSS}}$ (moveout) with increasing ray parameter (Fig. 9c). To measure $T_{\text{VDSS}}$ and $\Phi_{\text{VDSS}}$, we used a 20 s-window around $Ss$ as our source wavelet to account for the slightly lower frequency of the observed data compared with our synthetics. After measuring $T_{\text{VDSS}}$ and $\Phi_{\text{VDSS}}$, we excluded three ‘Grade B’ events with minimum misfit >0.4, because their large misfit implies that $SsPmp$ is poorly approximated by a phase-shifted source time function, violating a basic assumption. We used measured $T_{\text{VDSS}}$ and $\Phi_{\text{VDSS}}$ of the remaining seven ‘Grade A’ to derive $V_p^{av} = 6.7 \pm 0.2$ km s$^{-1}$, $H = 38 \pm 4$ km (Fig. 9e) and $V_p^{um} = \sim 8.0–8.1$ km s$^{-1}$ (Fig. 9f) as described above.

For PRF analysis at EDZN, we used a time-domain iterative deconvolution algorithm with a Gaussian bandwidth of 2.5 Hz (Herrmann 2013) to compute PRFs for teleseismic $P$-wave data from 30–90° (Fig. S5). We manually selected 73 high-quality radial RFs using the Funclab software package (Porritt & Miller 2018), moveout-corrected the RFs to normal incidence for primary $Ps$ conversions and stacked, which yields a clear Moho $Ps$ at 4.24 s and little energy from intra-crustal converters (Fig. 10a). Using...
Figure 9. $V_p^{um}$, $V_p^{av}$ and Moho depth $H$ near station YKW3 derived from $T_{VDSS}$ and $\Phi_{VDSS}$. (a) Station and event distribution on a polar plot centered on YKW3. 10 Grade A and B events with backazimuth 297–304°, $M_w>5.5$, distance 40–60° and depth 30–700 km, are coloured by their focal depth. (b) Locations of broadband stations YKW3 and EDZN (triangles), LITHOPROBE reflection and refraction profiles (blue line), and VDSS Moho reflection points and virtual sources. The reflection-point and virtual-source locations are computed using the best-fitting $V_p^{av}$ and $H$ (part e) for the seven Grade A events (part c). PRF results for EDZN are shown in Figs. 10 and S5. (c) $P$- and $S$-component waveforms of the ten Grade A and B events. Events are sorted by their ray parameters (small to large), with detail information in Table S1. Black arrow marks Event 6 shown in (d). (d) Observed and best-fitting $P$-component waveforms of Event 6. (e) Observed $T_{VDSS}$ as functions of ray parameter and the best-fitting model from linear regression. (f) Observed and theoretical relations between $\Phi_{VDSS}$ and ray parameter for variable $V_p^{um}$ but $V_p^{lc}$ fixed at 6.8 km s$^{-1}$. The observations favour $V_p^{um} \sim 8.0–8.1$ km s$^{-1}$. 

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estimate the uncertainties of RFs using a phase weight ratio in our VDSS analysis, we estimate $\kappa$ (Kanamori 2000) and $\kappa_{H\kappa}$ (Fig. 7 in Part 1). We also performed conventional $H$-$\kappa$ stacking of our seven Grade A events, making $\kappa_{H\kappa}$ insensitive to $H$ and $V_p^{\text{um}}$ from our VDSS analysis, we estimate $\kappa_{H\kappa} = 1.75 \pm 0.04$ (Fig. 10b).

We also computed transverse RFs. We found neither azimuthal variation of Moho $Ps$ on the radial RFs (Figs S5b and c) nor clear Moho $Ps$ on the transverse RFs (Figs S5d and e), indicating negligible Moho dip or crustal anisotropy. We do however observe a coherent negative arrival at $\sim 8$ s on the transverse RFs (Figs S5d and e), which may correspond to the intra-mantle ‘H converter’ imaged beneath Yellowknife by Bostock (1998).

$T_p$, measured from our moveout-stacked PRFs and $H$ and $V_p^{\text{um}}$ from our VDSS analysis, we estimate $\kappa_{H\kappa} = 1.75 \pm 0.04$ (Fig. 10b). We also performed conventional $H$-$\kappa$ stacking of our 73 selected RFs using a phase weight ratio $w_p \cdot w_{p\kappa} \cdot w_{p\kappa} = 1.05:0.5$ (Zhu & Kanamori 2000) and $V_p^{\text{um}} = 6.7$ km s$^{-1}$ taken from our VDSS analysis to determine $H_{H\kappa} = 37.6$ km and $\kappa_{H\kappa} = 1.75$ (Fig. 10c). We estimate the uncertainties of $H_{H\kappa}$ and $\kappa_{H\kappa}$ ($\pm 2.0$ km and $\pm 0.04$, respectively) using the range with stacking amplitude $> 95$ per cent of the peak value while ignoring the additional uncertainties arising from the choice of $V_p^{\text{um}}$ and phase stacking weights (supplementary figures in Karplus et al. 2019).

We also computed transverse RFs. We found neither azimuthal variation of Moho $Ps$ on the radial RFs (Figs S5b and c) nor clear Moho $Ps$ on the transverse RFs (Figs S5d and e), indicating negligible Moho dip or crustal anisotropy. We do however observe a coherent negative arrival at $\sim 8$ s on the transverse RFs (Figs S5d and e), which may correspond to the intra-mantle ‘H converter’ imaged beneath Yellowknife by Bostock (1998).

### 3.2 Uncertainties and sensitivities of VDSS analyses

The observed $T_{\text{VDSS}}$ of Grade A events clearly decreases with increasing ray parameter, with a typical uncertainty of $\pm 0.3$ s (Fig. 9e). We find the best-fitting crustal $V_p^{\text{um}}$ and Moho depth ($H_{\text{VDSS}}$) to be $6.7 \pm 0.2$ km s$^{-1}$ and $38 \pm 4$ km, respectively (Fig. 9e). The observed $\Phi_{\text{VDSS}}$ of Grade A events also shows a clear decrease with increasing ray parameter (Fig. 9f), in accord with our theoretical predictions (Part 1). To find the best-fitting $V_p^{\text{um}}$, we plot theoretical $\Phi_{\text{VDSS}}$-P relations for different $V_p^{\text{um}}$ values while fixing $V_p^{\text{lc}} = 6.8$ km s$^{-1}$, the value given by wide-angle refraction studies close to our VDSS reflection points (Fig. 9f, Fernández-Viejo & Clowes 2003). Comparing our observed values with the theoretical relations, we find that the observations favour $V_p^{\text{um}} = 8.0$–8.1 km s$^{-1}$, consistent with the $Pn$ velocity of $8.2 \pm 0.1$–0.2 km s$^{-1}$ reported by the wide-angle refraction studies (Fernández-Viejo & Clowes 2003; Fernández-Viejo et al. 2005). Our choice of $V_p^{\text{lc}}$ has little effect on estimated $V_p^{\text{um}}$ given the $\sim 20$° uncertainty in observed $\Phi_{\text{VDSS}}$, as shown by theoretical curves computed assuming $V_p^{\text{lc}} = 6.5$, 6.8 and 7.1 km s$^{-1}$ ($\sim 4$ per cent perturbation) (Fig. 11). This is because the range of $1/p$ considered here (7.5–8.0 km s$^{-1}$) is significantly higher than the assumed $V_p^{\text{lc}}$, making $\Phi_{\text{VDSS}}$ insensitive to $V_p^{\text{lc}}$ (Fig. 7 in Part 1).

In addition to using only our seven Grade A events, we test the effects of also including Grade B and C events, which gives significantly higher values for $V_p^{\text{um}}$ and $H$ (Fig. S6a). This increase in $V_p^{\text{um}}$ and $H$ is primarily due to the large $T_{\text{VDSS}}$ of Grade B and C events with small ray parameter (Fig. S6a). Similarly, inclusion of
Grade B and C events significantly increases our estimate of $V_{p}^{um}$ (Fig. S6b). This increase in $V_{p}^{aw}$, $H$ and $V_{p}^{um}$ is primarily due to most of the Grade B and C events coincidentally having smaller ray parameters than Grade A events, even though our data selection was not based on ray parameter. For events with $p < 0.122$ s km$^{-1}$, the turning velocity $1/p$ of the downgoing $P$ wave exceeds 8.2 km s$^{-1}$, higher than the velocity we estimated for the top of the upper mantle ($V_{p}^{um} = 8.1$–8.0 km s$^{-1}$). Therefore, $SsPmp$ with $p < 0.122$ s km$^{-1}$ has turned or been refracted within the upper mantle below the CMB. These events thus likely provide estimates of $V_{p}^{um}$ more appropriate for the depth at which the rays turn, and estimates of $V_{p}^{aw}$ and $H$ that average the crust and upper mantle. Previous wide-angle refraction results suggested that $V_{p}$ increases from 8.2 s km$^{-1}$ right below the Moho to 8.5 km s$^{-1}$ at ~65 km depth between YKW3 and EDZN (Fernández-Viejo & Clowes 2003), consistent with our observation that $\Phi_{VDSS}$ at low ray parameter suggests $V_{p}^{um} = 8.4$–8.6 km s$^{-1}$ (Fig. S6b). Additional difficulties may arise if the turning velocity is only slightly higher than the velocity just below a sharp CMB, because strong pre-critical reflections at the CMB will interfere with $SsPmp$ that turns in the upper mantle (Fig. 5 in Part 1), causing a complicated $SsPmp$ waveform that cannot be modeled with a phase-shifted $S$ wavelet. Moreover, as $p$ decreases and $SsPmp$ becomes a turning wave in the upper mantle, the distance between the virtual source and station grows, increasing the effects of lateral heterogeneity on $T_{VDSS}$ and $\Phi_{VDSS}$. These complications may cause the large uncertainty and scatter of measured $T_{VDSS}$ and $\Phi_{VDSS}$ for events with small $p$ (Fig. S6). In summary, when constraining $V_{p}^{aw}$ and $H$ with $T_{VDSS}$, it is appropriate to only use events with $1/p$ smaller than $V_{p}^{um}$, because the behaviour of $SsPmp$ as an upper-mantle turning wave is more complicated than modelled here. Nonetheless, this behaviour may open the way to continuous depth profiling of upper-mantle velocity structure in future work.

3.3 Comparison with previous imaging and geologic interpretations

Our Moho depth ($H_{VDSS} = 38 \pm 4$ km, Fig. 9e) is in good agreement with LITHOPROBE near-vertical reflection results (~37 km), but slightly deeper than their seismic refraction results (~31–33 km, Fernández-Viejo et al. 2005; Hammer et al. 2010, Fig. 12). Our average crustal $V_{p}$ ($V_{p}^{av} = 6.7 \pm 0.2$ km s$^{-1}$, Fig. 9e) is also higher than the value given by the wide-angle refraction studies (~6.4 km s$^{-1}$, Fernández-Viejo et al. 2005; Hammer et al. 2010, Fig. 12). Our $H_{VDSS}$ closely matches our $H$–$\kappa$ Moho depth ($H_{VDSS} = 38.2 \pm 3.8$ km; $H_{VDSS} = 37.6 \pm 2.0$ km; Fig. 10c). This agreement not only verifies our VDSS method, but also indicates that $V_{p}^{aw} = 6.7$ km s$^{-1}$ assumed for the $H$–$\kappa$ analysis is likely a reliable estimation of average crustal $V_{p}$. Similarly, the agreement between $\kappa_{joints} = 1.74 \pm 0.06$ and $\kappa_{H–H}$ = 1.75 $\pm$ 0.04 (Fig. 10) further demonstrates the robustness of our method, though our $\kappa$ values are higher than those from the refraction study that found $\kappa = 1.68$–1.72 in the Slave Craton (Fernández-Viejo et al. 2005).

Our joint distribution between $V_{p}^{aw}$ and $\kappa_{joints}$ shows a clear negative correlation (Fig. 13), similar to our synthetic example (Fig. 8), though the field data is less focused than the synthetics due to larger observation uncertainty. We again compare the joint distribution with the temperature-corrected laboratory measurements from Christensen (1996) and Hacker et al. (2015), which implies an intermediate (dioritic) average crustal composition, though some mafic rock samples also fit our preferred $V_{p}^{aw}$–$\kappa$ estimates (Fig. 13). The refraction $V_{p}^{aw}$ and $\kappa$ values (Fernández-Viejo et al. 2005) are quite distinct (green ellipse in Fig. 13) and imply a felsic (granitic) average crustal composition.

Fernández-Viejo et al. (2005) interpreted their low average $V_{p}^{aw}$ and $\kappa$ as evidence for a ‘significant felsic content’ of the crust in the SW Slave Craton and explained the regional Bouguer gravity low with their silicic but thin crust. Our more-mafic but thicker crust could equally match the gravity signature. We also note that the lower $V_{p}^{aw}$ and thinner crust found by wide-angle refraction experiments could partly be explained by a positive trade-off between the two parameters.

The Slave craton is home to the Earth’s oldest rocks, the Acasta Gneiss with Hadean ages $\geq$4 Ga (Bowring & Williams 1999), though the bulk of its crust is clearly Archaean. In the SW Slave Craton, geochronology of surface exposures emphasizes the importance of Neoarchaean crustal growth at $<2.6$ Ga (Bennett et al. 2005), but other crustal-scale cross-sections show the Palaeoarcheans-to-Mesoarchean Central Slave Basement Complex forming the bulk of the crust (Bleeker 2002), as also interpreted from LITHOPROBE near-vertical reflection profiling (van der Velden & Cook 2002). Our observation of crustal thickness, $V_{p}^{aw}$ and $\kappa$ may help resolve the dominant crustal formation age of the SW Slave Craton. A trend of increasing $V_{p}^{aw}$, $\kappa$ and crustal thickness with decreasing crustal age in the Western Australia Craton has been suggested to reflect a global transition of crust-forming mechanism from plume tectonics to plate tectonics during Archean (Yuan 2015). If this relationship were to be applicable to the Canadian cratons, our results are most consistent with a Mesoarchean formation age for the bulk crust of the SW Slave Craton.

4 DISCUSSION

Post-critical $SsPmp$ has three major attributes: $T_{VDSS}$, $\Phi_{VDSS}$ and $A_{VDSS}$. In 1-D, $A_{VDSS}$ is largely affected by near-surface velocity (Fig. 10 in Part 1), whereas $T_{VDSS}$ and $\Phi_{VDSS}$ both contain information on crustal and upper-mantle structure. In this paper, we propose methods to retrieve Moho depth, $V_{p}^{aw}$ and $V_{p}^{um}$ from $SsPmp$ observations under a 1D assumption.

$T_{VDSS}$ is controlled by Moho depth and $V_{p}^{aw}$ and can be measured together with $\Phi_{VDSS}$ by waveform fitting (Fig. 5b). Whereas Moho depth can be estimated from a single $SsPmp$ observation with an assumed $V_{p}^{aw}$ (Fig. 4), both $T_{VDSS}$ and $V_{p}^{aw}$ can be constrained using multiple events recorded at the same station (Fig. 5) (Kang et al. 2016). The uncertainties of our derived $H_{VDSS}$ and $V_{p}^{aw}$ at YKW3 (4 km and 0.2 km s$^{-1}$, respectively) are larger than those given by Kang et al. (2016) for two stations FORT and WB2 in Australia (~3.0 km and ~0.15 km s$^{-1}$, respectively). Two factors might contribute to the higher uncertainties of our measurements. First, the uncertainty of our $T_{VDSS}$ (~0.3 s) is greater than Kang et al. (~0.18 s), but these values are not directly comparable because we compute $T_{VDSS}$ uncertainties quantitatively (Supplementary Text 1), whereas Kang et al. (2016) estimated their uncertainties empirically. Second, the ray-parameter range of our study (0.1247–0.1327 s km$^{-1}$) is smaller than in Kang et al. (0.1223–0.1349 s km$^{-1}$ for FORT and 0.1226–0.1369 s km$^{-1}$ for WB2), largely because the number of events we used (seven) is smaller than in Kang et al. (20 for FORT and 12 for WB2). Our events are fewer and our ray-parameter range smaller in part due to our strict data-selection criteria (Fig. S6). In practice, the number of observations available depends on event distribution and data quality.

$\Phi_{VDSS}$ is controlled by $V_{p}^{lc}$ and $V_{p}^{um}$ and can be used to constrain the two parameters. Due to the trade-off between $V_{p}^{lc}$ and
Post-critical SsPmp and its applications to VDSS, Part 2

Figure 12. Comparison between our VDSS and PRF and LITHOPROBE reflection and refraction results. The migrated reflection image is overlain on the refraction $V_p$ model (after Hammer et al. 2010). Red circle with error bars: VDSS Moho plotted at the reflection point; the arrows are ray paths of $SsPmp$. Yellow curve: moveout-stacked PRF mapped to depth domain and plotted at EDZN. VDSS, PRF and reflection Moho depths agree with each other (~38 km), but the refraction Moho is shallower (~34 km).

Figure 13. Average crustal composition of the SW Slave Craton constrained with PRF-VDSS joint analysis. The probability density function is derived from 5000 random simulations of $V_p^av$ and $V_p/V_s$ ratio. The diamonds and circles are the same as Fig. 8. The orthogonal trends of the probability density function and rock properties together constrain average crustal composition. Green ellipse is $V_p^av$ and $\kappa$ from Fernández-Viejo et al. (2005).

$V_p^um$ (Fig. 7 in Part 1), it is impossible to determine one without knowing the other (Fig. 3). However, we show that when $p$ is small (1/$p$ close to $V_p^um$), $\Phi_{VDSS}$ depends only weakly on $V_p^k$, allowing determination of $V_p^um$ without a precisely known $V_p^lc$ (Figs 3 and 4; also Fig. 7 in Part 1). Although $V_p^um$ can in principle be derived from a single $SsPmp$ observation, incorporating $\Phi_{VDSS}$ from multiple events offers a more robust estimation of $V_p^um$, especially in the presence of noise (Figs 5 and S4). We note that, when measuring $\Phi_{VDSS}$, it is important to choose traces with high signal-to-noise ratio, as waveform observables are more sensitive to noise than traveltime measurements. This is especially true when considering the higher noise level of teleseismic $S$ compared to $P$ wave due to coda waves generated by preceding $P$ phases that arrive in the same time window. A practical criterion for data quality control is to choose $SsPmp$ traces that can be well-fitted with a phase-shifted $Ss$ wavelet (Fig. 9). Our field examples show that $\Phi_{VDSS}$ and $T_{VDSS}$ of events selected this way (cyan markers in Fig. S6) in general have smaller uncertainties than the events not selected (grey markers in Fig. S6). While inferring $V_p^um$, $H_{VDSS}$ and $V_p^av$ from $\Phi_{VDSS}$ and $T_{VDSS}$ observations, we implicitly assume the CMB to be a sharp boundary as opposed to a broad transition zone. When the CMB is a velocity-gradient zone thinner than the dominant wave-length of the incident $P$ wave (~25 km for typical field data), as is true for most continental areas, $\Phi_{VDSS}$ is not significantly different from the case with a sharp CMB (Part 1), thus our method of deriving $V_p^um$ from $\Phi_{VDSS}$ still applies. When the CMB is a velocity-gradient zone with $SsPmp$ rays turning within the CMB (1/$p$ in the range of the gradient-zone $V_p$), the $T_{VDSS}$-$p$ relation (moveout) is different from the case with a sharp CMB (Fig. 9 in Part 1), making our method of deriving $H_{VDSS}$ and $V_p^av$ from $T_{VDSS}$ less accurate. However, the very good agreement between $H_{VDSS}$ and $H_{Hg}$ (Fig. 10c) for the SW Slave Craton not only demonstrates the robustness of our method, but also verifies the underlying assumptions that we make about the study area. First, the study area is well approximated with a 1-D lithospheric model. Secondly, the CMB beneath the study area is
sufficiently sharp that SsPmp reflections and Ps conversions happen at essentially the same depth.

5 CONCLUSIONS

We have shown with synthetic and field examples that $T_{\text{VDSS}}$ and $\Phi_{\text{VDSS}}$ can be used to derive average crustal $V_p$, crustal thickness and $V_p$ of the uppermost mantle. We have also demonstrated that average crustal $V_p/V_s$ ratio and composition can be estimated with PRF-VDSS joint analysis. Our field example from the SW Slave Craton shows an intermediate crustal composition, together with crustal thickness suggestive of a Mesoarchean formation age.

ACKNOWLEDGEMENTS

We thank Brad Hacker from University of California, Santa Barbara, who provided his compilation of laboratory measurements. We thank the Data Management Center of the Incorporated Research Institutions for Seismology (IRIS-DMC) for the waveforms of the Yellowknife array and Canadian Hazards Information Service for the data of the POLARIS array. We also thank Jieyuan Ning from Peking University and two anonymous reviewers for their valuable advice. Tianze Liu is supported by a Stanford Graduate Fellowship. Gabriel Ferragus was supported by an IRIS internship during his stay at Stanford.

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SUPPORTING INFORMATION

Supplementary data are available at GJI online.

Text S1. Estimating uncertainties of $\Phi_{\text{VDSS}}$ and $T_{\text{VDSS}}$.
Table S1. Information of Grade A and B events recorded at YKW3.
Figure S1. Constraining Moho depth and $V_p$ using synthetic data with 10 per cent white noise.
Figure S2. Distribution of $V_p^{um}$ derived from random simulations of noisy synthetic SsPmp waveforms.

Figure S3. Uncertainty estimation for $\Phi_{VDSS}$.

Figure S4. Constraining Moho depth, $V_p^{m}$ and $V_p^{um}$ using synthetic data with 10 per cent white noise.

Figure S5. PRF observations for EDZN.

Figure S6. Effects of data selection on estimated Moho depth, $V_p^{m}$ and $V_p^{um}$, for real data.

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